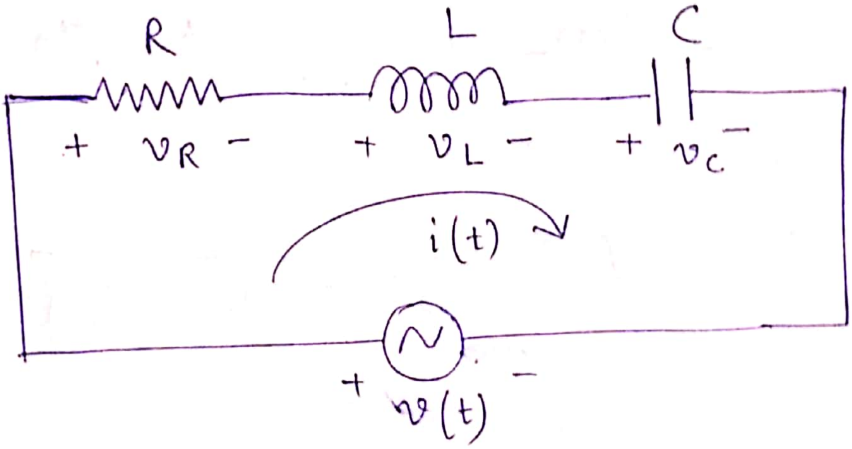
SEM II Physics Hons.

Paper CC-III

Electricity and Magnetism

Series LCR circuit:

① Resonance, ② Power Dissipation ~~and~~ ③ Quality factor and ④ Band width.



$v(t)$, v_R, v_L & v_C are the instantaneous voltages and $i(t)$ is the instantaneous current.

Applying KVL for the ckt

$$v_R(t) + v_L(t) + v_C(t) = v(t)$$

or,
$$Ri + L\frac{di}{dt} + \frac{q}{C} = V_0 e^{j\omega t}$$

Differentiating the above eqn. w.r.t. t

or,
$$R\frac{di}{dt} + L\frac{d^2i}{dt^2} + \frac{1}{C}\frac{dq}{dt} = j\omega V_0 e^{j\omega t}$$

Let the steady state solution for the current is $i = Ae^{j\omega t}$

Then, $\frac{di}{dt} = j\omega A e^{j\omega t}$

$$\frac{d^2 i}{dt^2} = -\omega^2 A e^{j\omega t}$$

$$\left(R.j\omega - \omega^2 L + \frac{1}{C}\right) A e^{j\omega t} = j\omega V_0 e^{j\omega t}$$

or $\left(j\omega R - \omega^2 L + \frac{1}{C}\right) A = j\omega V_0$

or $\left(R + j\omega L + \frac{1}{j\omega C}\right) A = V_0$

or, $A = \frac{V_0}{R + j\left(\omega L - \frac{1}{\omega C}\right)}$

$$\therefore \quad i = \frac{V_0 \quad e^{j\omega t}}{R + j\left(\omega L - \frac{1}{\omega C}\right)}$$

Where, $R + j\left(\omega L - \frac{1}{\omega C}\right) = Z$ is the impedance of the circuit.

putting $R = B\cos\phi$ & $\omega L - \frac{1}{\omega C} = B\sin\phi$

we can write

$$Z = B e^{j\phi} = |Z| e^{j\phi}$$

where $|Z| = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$

& $\tan\phi = \frac{\omega L - \frac{1}{\omega C}}{R}$

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$$\therefore \quad i = \frac{V_0}{|Z|} e^{j(\omega t - \phi)}$$

putting $I_0 = V_0/|Z|$

and using rms values as

$$|I| = \frac{I_0}{\sqrt{2}} \quad \& \quad |V| = \frac{V_0}{\sqrt{2}}$$

$$I_0 e^{j(\omega t - \phi)} = \frac{V_0 e^{j\omega t}}{|Z| e^{+j\phi}}$$

or $$|I| e^{-j\phi} = \frac{|V_0|}{|Z| e^{j\phi}}$$

or $$\boxed{V = Z I}$$

$$V = \left[R + j\left(\omega L - \frac{1}{\omega C}\right)\right] I$$

$$V = RI + j\omega L I - \frac{j}{\omega C} I$$

RI is the phasor for V_R

$j\omega L I$ is the phasor for V_L

$-\frac{jI}{\omega C}$ is the phasor for V_C

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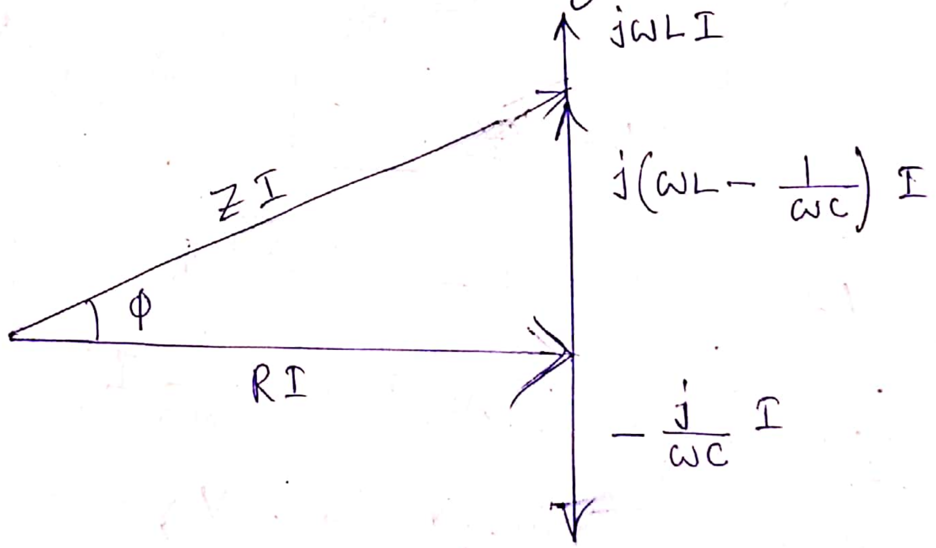
The phasor diagrams for the following 3 possible cases.

Case I : when $\omega L > \frac{1}{\omega C}$

i.e. the inductive reactance > capacitive reactance.

$\therefore$ By $\tan\phi = \frac{\omega L - \frac{1}{\omega C}}{R} = +ve$

i.e. current is lagging behind the applied voltage by an angle ϕ and the corresponding phasor diagram

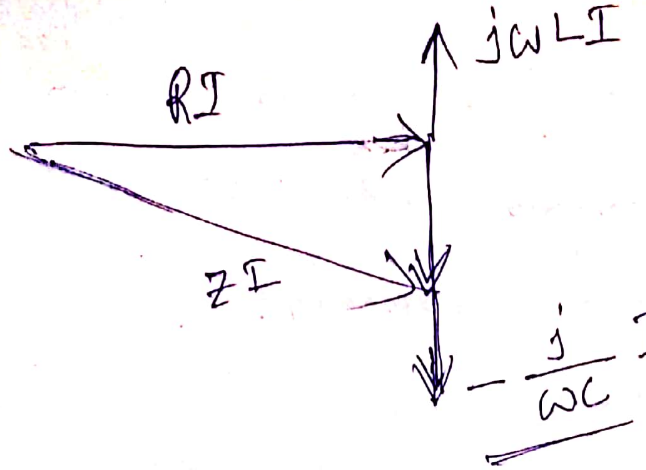


Case 2 : when $\omega L < \frac{1}{\omega C}$

i.e. inductive reactance < capacitive reactance

$\therefore$ ϕ is negative

i.e. the current leads the applied voltage by an angle ϕ

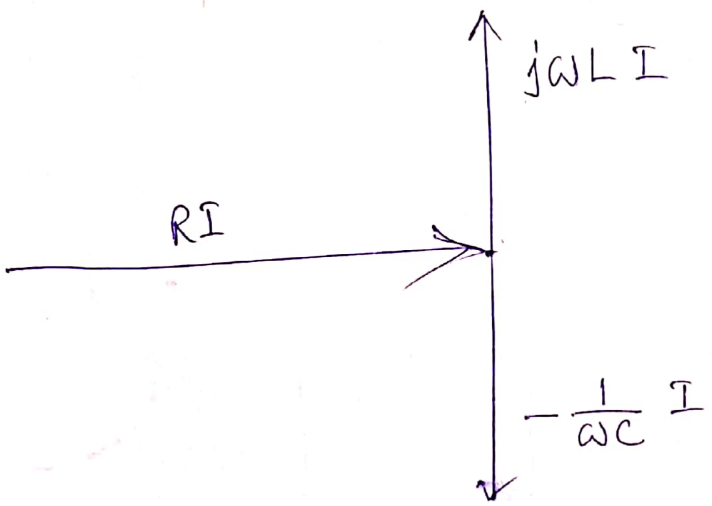


Case 3: When $\omega L = \frac{1}{\omega C}$

i.e. Reactance becomes zero

i.e. the ckt is purely resistive.

The current will be in phase with the applied voltage, as $\phi = 0$



Resonance: In a series LCR circuit when the applied voltage and the current are in phase, ie. when the circuit is purely resistive we call it series resonance.

from $\omega L = \frac{1}{\omega C}$

$$\omega = \omega_0 = \frac{1}{\sqrt{LC}}$$

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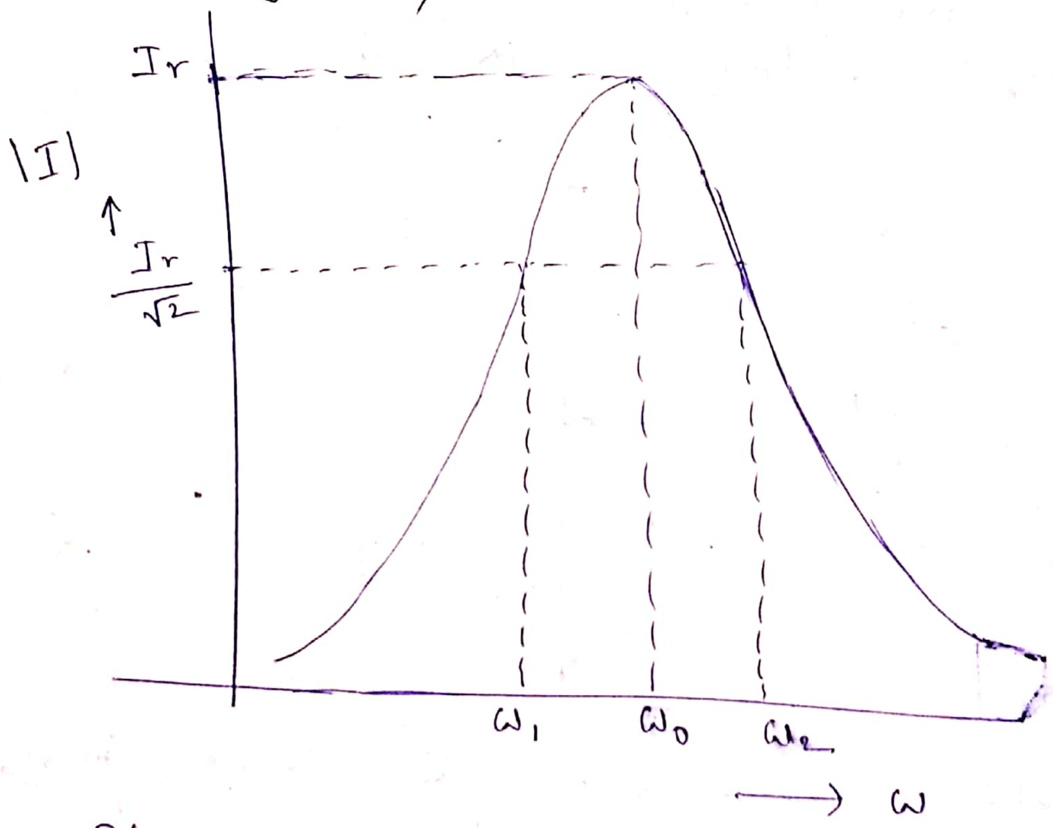
$\therefore$ $f_0 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi\sqrt{LC}}$ denotes the resonant frequency and ω_0 is called resonant angular frequency.

We already have,

$$|Z| = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

Since $\left(\omega L - \frac{1}{\omega C}\right)^2$ is +ve, $|Z|$ is greater than R for all off-resonant frequencies and equals to R at resonant frequency.



If ω_x be the angular freq. at which the current becomes $I_r/\sqrt{2}$

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$$\therefore \frac{I_r}{\sqrt{2}} = \frac{V}{\sqrt{2}R} = \frac{V}{\left[R^2 + \left(\omega_x L - \frac{1}{\omega_x C}\right)^2\right]^{1/2}}$$

$$\Rightarrow \quad 2R^2 = R^2 + \left(\omega_x L - \frac{1}{\omega_x C}\right)^2$$

or, $\omega_x^2 LC \mp \omega_x RC - 1 = 0$

$\therefore$ The two +ve roots of ω_x are

$$\omega_1 = \frac{\sqrt{R^2C^2 + 4LC} - RC}{2LC}$$

and $$\omega_2 = \frac{\sqrt{R^2C^2 + 4LC} + RC}{2LC}$$

$\therefore$ The band width $\omega_2 - \omega_1 = \frac{R}{L}$.

The Q-factor (quality factor)

$$Q = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 CR} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

$$\therefore \omega_2 - \omega_1 = \frac{\omega_0 R}{\omega_0 L} = \frac{\omega_0}{Q}$$

$$\therefore \boxed{Q = \frac{\omega_0}{\omega_2 - \omega_1}}$$

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